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Distributed Coordination of Asymmetric Compartmental Systems Under Time-Varying Inputs and Its Applications

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***Abstract*— In this paper, we present new distributed coordination for a set-point regulation of asymmetric compartmental systems under periodically time-varying external inputs. The feature of the proposed coordination is to adjust system parameters, which is directly related to interflow rate among compartments and outflow rate to environment. In contrast to symmetric compartmental systems, there is a difficulty in analysis resulted from a property of compartmental matrix in the asymmetric cases; however, we solve the problem by designing an appropriate coordination range of the system parameters. In order to compensate the time-varying effect in the inputs, we assume that the structure of the inputs is a finite Fourier series, and show that the coordination of the system parameters should have the same structure with the same frequencies. The remainder of the coordination is to determine coefficients of the Fourier series of the system parameters. We develop a distributed update law for obtaining the coefficients. We apply the distributed coordination to water irrigation systems and water distribution systems, and show the effectiveness of the proposed distributed coordination schemes through numerical simulations.**

***Index Terms*— Compartmental systems, distributed coordination, set-point regulation, time varying inputs, water distribution systems, water irrigation systems.**

I. Introduction

COMPARTMENTAL systems feature attribute (or energy) distribution or transportation in complex networks; therefore, these are modeled in various applications such as pharmacokinetics, chemical reaction, ecology [1], [2], epidemics [3], air traffic network [4], contaminant transport systems [5], cancer chemotherapy [6], distillation columns [7], water tank system [8], water irrigation systems [9], water distribution systems [10], and so on.

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Some mathematical properties of the compartmental systems have been studied. For example, all the states of the compartmental systems are nonnegative for all nonnegative initial conditions and input values [11]. Regarding stability property, all eigenvalues of the compartmental matrix of the compartmental systems are nonpositive. Especially, Fife [12] and Foster and Jacquez [13] have shown that the number of zero eigenvalues is equal to that of simple traps. Reachability and observability of the compartmental systems have been studied in [14], and condition of nonoscillatory solution has been presented in [15]. In [16] and [17], positive realization problem, i.e., the existence of realization for the transfer function of the compartmental system, has been analyzed.

There also have been synthesis problems of the compartmental systems. Bastin and Guffens [18] have raised the congestion problem of the compartmental systems, and have suggested a nonlinear output feedback controller preventing the congestion. In [19], a positive observer for the compartmental system has been presented. A model reference tracking via H_∞ control and sliding mode control has been studied in [20] and [21], respectively. As a set-point regulation problem, Bastin and Provost [22] have defined the set point as total state values, and proposed its positive feedback controller. The set point can be defined as the desired state of each compartment. With this definition, a distributed adjustment of external inputs of each compartment for the set-point regulation and its extension to noisy measurements have been studied in [11] and [23], respectively. Alternatively, in [24], the set-point regulation is achieved by the coordination of system parameters, which are related to interflow rate among compartments and outflow rate.

In this paper, we further discuss the distributed coordination scheme of [24]. It is notable that the result of [24] is limited to symmetric linear compartmental systems in which the interconnection graph is undirected and system parameters are symmetric. It is necessary to consider the asymmetrical case in order to extend applicability. Furthermore, one of underlying assumptions on the result of [24] is that the external inputs are fixed as constant. However, the inflow rate can be time-varying in real situations. For example, in water distribution systems [10], the external inputs are water supply from fresh water resources, and the amount of water supply